[coins & candy... and options]

Carolina Poker Club: Problem Set 3

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September 2024

1 Coins & Candy

1. Using one fair coin, how can you simulate a $\frac{1}{3}$ random variable?

   Problems 2-5 involve the following game:

   Suppose you are playing a game in which you begin with 3 pieces of candy. The rules of the game involve a flip of a coin and are the following:

   - If you flip heads, you win 3 pieces of candy
   - If you flip tails, your current candy amount gets divided by three (yes, the candy may be cut into non-integer pieces)

2. If you are given the option to flip the coin twice or not at all, should you do so?

3. Suppose you did not start with 3 pieces of candy. How many pieces of candy should you begin with to make you indifferent to flipping the coin once?

4. Now say you start with 81 pieces of candy and you were to flip the coin infinitely many times. What would your average balance be?

5. Again, you start with 81 pieces of candy. You are given the option to flip the coin as much or as little as you would like. Should you flip the coin at all? Defend your answer with an explanation.

2 Options Volatility & Pricing

Problems 1-3 involve the following data:

Imagine you know the probability distribution of a stock on Jan 1, 2025 for Jan 25, 2025:

- 10% chance of being worth $180.
- 30% chance of being worth $150.
- 20% chance of being worth $210.
- 40% chance of being worth $90.

1. What's the intrinsic price of the Jan '25 $200 strike call option?

2. Would the March '25 call of the same strike be worth more or less?

3. What's the intrinsic value of the Jan '25 $100 put option?

3 Solutions

3.1 Coins & Candy

1. Begin by flipping the coin twice. There are four outcomes: HH HT TH TT. Assign one of these four outcomes to your first variable, two of these outcomes to your second variable, and assign the last outcome to repeat the methods above. This gives a recursive solution that correctly simulates a probability of $\frac{1}{3}$

2. You should. The expectation of the game may be written as:

$$E_{n+1} = \frac{1}{2}(\frac{E_n}{3}) + \frac{1}{2}(E_n + 3)$$

   By using E_2 and E_1 we find

   $E_2 = \frac{1}{2}(\frac{1}{2} * \frac{3}{3^2} + \frac{1}{2} * (1+3)) + \frac{1}{2}(\frac{1}{2} * (3+3+3) + \frac{1}{2} * \frac{3+3}{3}) = \boxed{\frac{23}{6} > 3}$

3. Similarly to the previous problem, we just need to solve the expectation $E_{n+1} = \frac{1}{2}(\frac{E_n}{3}) + \frac{1}{2}(E_n + 3)$. It is important to realize that, when you are indifferent to playing the game, the expected candy balance of your next flip should be the same as your current candy balance. Solving for your balance gives
   $E_n = \frac{1}{2}(\frac{E_n}{3}) + \frac{1}{2}(E_n + 3) \implies E_n = \boxed{\$4.5}$

4. Intuition check. Flipping infinitely should take our average to our indifference balance. $\boxed{\$4.5}$

5. Argue your answer. Yes: Although you are losing expected value on your first flip, the ability to flip indefinitely allows you to eventually flip heads more than 27 times in a row, giving you more than 81 pieces. No: Impractical. Flipping until you get back to 81 would happen requires 27 heads in a row, which happens, on average, once every $2^{27} = 134,217,728$ flips. Nobody can flip a coin this much!

3.2 Options Volatility & Pricing

1. The intrinsic value of an option is $0 below the strike price and grows at a rate of y = x once the underlier reaches the strike price. For example, if a stock was trading for $50, the $55 strike option would be worthless (intrinsically) since it's cheaper to buy the stock on the market for $50 than to exercise the option and buy it for $55. But the $45 strike option would be worth $5 (or $500 with the typical 100x multiplier on stocks) since it lets you buy stock for a discount of $5 per share. As such, the expected value of an option is the probability weighted average of the individual intrinsic values. The computations give
   $0 + 0.2 * \$10 = \$2$, or $200 with the typical multiplier.

2. When looking at total worth, we look at the intrinsic value (which is the same) and the extrinsic value, also known as the time premium. Options with a longer expiration have more "optionally", and thus have a longer time premium, so the March option would be worth more.

3. Applying the same logic as problem 1), we get 0 + 0.4*10 = $4, or $400 with the typical multiplier.